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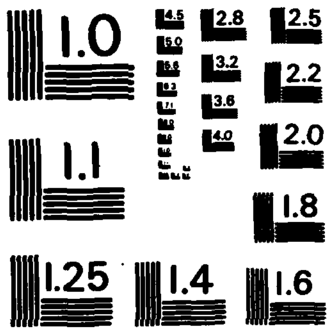

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A Birth and Death Process Approximation For the Slotted ALOHA Algorithm

INTRODUCTION

Many authors have concerned themselves with the bistable behavior of the finite-user slotted ALOHA protocol under heaving loading. Recently Neslon used a catastrophe-theoretic approach to demonstrate that under a fluctuating load the protocol suffers hysteresis as well as bistability. He uses results from catastrophe theory to give a possible improved control algorithm.

Central to Nelson's approach is a diffusion approximation of the queue of backlogged users. This approximation has the advantage of yielding a continuing probability density for the process, thus allowing the use of (stochastic) catastrophe throry. Unfortunately, as will be seen later, the approximation requires difficult numerical integration and yields no closed form solution.

It is being proposed here that the process should remain discrete, and that it can be approximated reasonably well as a birth-death process. This allows rapid computation of the approximate stationary distribution.

THE SLOTTED ALOHA MODEL

Assume there are N users, each of whom wishes to send packets across the communications channel at various times with equal probability. Time is divided into equal slots, and all packets are no bigger than one time slot. Note that if two or more users attempt to use the channel at once,

their packets collide (i.e. become garbled), and no packet is transmitted successfully. These packets must still be transmitted at some future time; hence users of previously collided packets are referred to as backlogged. Those users who are not backlogged are called <u>idle</u>. Using these labels the ALOHA protocol can be defined. At the beginning of each slot each idle user receives a packet to transmit with probability p_0; these newly arrived packets (arrivals) are immediately transmitted; each backlogged user transmits its packet with probability p_1; In case of a successful transmission, the user reverts to being idle. Note that the users are independent of one another, and their transimission attempts are independent across time. It should also be noted that by definition a backlogged user has <u>only one packet</u> which it must transmit; buffering is <u>not</u> allowed.

This model can readily be modelled by a Markov chain X_t on $0,1,\ldots,N$ in which the states of the process represent the number of backlogged users. The transmission probabilities $P_{ij} \triangleq Pr\{X_{t+1} = j | X_t = i\}$ are given below:

$P_{00} = (1 - p_0)^N$:	empty queue, no arrivals
$P_{01} = 0$:	empty queue, if one packet arrives it is sent successfully
$P_{0k} = \binom{N}{k} p_0^k (1-p_0)^{N-k} \quad 2 \le k \le N$:	empty queue, collisions
$P_{n,n-k} = 0 \quad k \ge 2$:	no more than one packet can be sent at a time
$P_{n,n-1} = (1-p_0)^{N-n} n\, p_1(1-p_1)^{n-1}$:	no arrivals, one line transmission
$P_{n,n} = (1-p_0)^{N-n}(1-np_1(1-p_1)^{n-1})$:	either no arrivals and no successful line transmissions or
$+(N-n)p_0(1-p_0)^{N-n-1}(1-(1-p_1)^n)$:	one arrival which is successfully transmitted
$P_{n,n+1} = (N-n)p_0(1-p_0)^{N-n-1}(1-(1-p_1)^n)$:	one arrival which is blocked by line transmissions
$P_{n,n+k} = \binom{N-n}{k} p_0^k (1-p_0)^{N-n-k}$:	more than one arrival.

These transition probabilities make for a fairly complitated Markov chain. The only "nice" property is that all the entries below the subdiagonal are zero (by condition (4).

NELSON'S APPROXIMATION

It is common to assess the stability or instability of an infinite Markov chain by looking at the drift and variance of each state (Lamperti 1960). These are defined as the expected jump and the expected squared jump:

$$d_i \stackrel{\Delta}{=} E(X_{t+1} - X_t | X_t = i) = \sum_j (j - i)P_{ij}$$
$$v_i \stackrel{\Delta}{=} E((X_{t+1} - X_t)^2 | X_t = i) = \sum_j (j - i)^2 P_{ij}$$

for finite chains, such as the one under consideration, stability is of no consequence, but the qualitative behavior of the chain can be assessed by finding the places at which the drift changes sign.

As Nelson states, the drift and variance for the N-user slotted ALOHA are given by

$$d_i = (N-i)p_0 - (S_1(i) + S_0(i))$$
$$v_i = (N-i)p_0((N-i)p_0 + (1-p_0)) + S_1(i) - S_0(i)$$

where

$$S_0(i) = (N-i)p_0(1-p_0)^{(N-i-1)}(1-p_1)^i$$
$$S_1(i) = ip_1(1-p_1)^{(i-1)}(1-p_0)^{(N-i)} \; .$$

He approximates the behavior of the queue by defining a diffusion process on (0,N) by taking the same functional form for the drift and variance, but allowing i to be real valued (instead of integer valued). To make

the distinction clear, the real valued functions will be denoted as $\mu(x)$ and $\sigma^2(x)$. He then defines the c-drift function as $\frac{2\mu(x)}{\sigma^2(x)}$ and the c-potential function as $V(x) \overset{\Delta}{=} \int^x \frac{2\mu(s)}{\sigma^2(s)} ds$. (These are are denoted as "c-drift" and "c-potential" to make the connection of the drift and potential functions used in catastrophe theory while keeping the stochastic nature of the model distinct from its deterministic analog). It should be boted that $v(x) = -\ell n\, s(x)$ where $s(x)$ is the scale density of the diffusion process.

Using c-drift and c-potential, Nelson shows that the slotted ALOHA can be viewed as a stochastic cusp catastrophe. He also approximates the stationary distribution of the discrete valued Markov chain with the stationary density of the related diffusion:

$$\chi(x) = ce^{-(v(x) + \ell n \sigma^2(x))}$$

where c is the normal constant.

This approach is very nice in its theoretical simplicity, since the catastrophe theoretic approach is interested only in the <u>shape</u> (i.e. location of the maxima, minima, and inflection points) of the stationary density $\chi(x)$. These can all be found by differentiating the c-drift function. Unfortunately finding error bounds for the final approximation of the stationary distribution by $\chi(x)$ appears to be difficult. This is the case because $v(x)$ has no closed form and can be found only via numerical integration. It is for this reason that the following material is being proposed.

BIRTH-DEATH APPROXIMATION

In order to ease the computations while keeping the flavor of Nelson's work, it is proposed here that the approximation of the slotted ALOHA should be kept in a discrete state-space. To do this the chain is approximated with a birth-death chain which can be considered in continuous or discrete time. The transition probabilities are defined so that the drift and variance of each interior state equal those of the original chain. To do this, define:

$$\lambda_i = \frac{1}{2}(v_i + d_i)$$
$$\mu_i = \frac{1}{2}(v_i - d_i)$$
$$\lambda_0 = d_0 \qquad \mu_2 = 0$$
$$\lambda_N = 0 \qquad \mu_N = d_N$$

where the λ's are the entries on the superdiagonal and the μ's are the entries on the subdiagonal of the transition (generator) matrix of the discrete (continuous) time birth-death chain. Note that with these definitions the d_0 indeed match the drift and variance:

$$E(X_{t+1} - X_t | X_t = i) = \lambda_i - \mu_i = d_i \qquad \text{(discrete time)}$$
$$E((X_{t+1} - X_t)^2 | X_t = i) = \lambda_i + \mu_i = v_i$$

(The continuous time calculations are the same)

Unfortunately, only the drift can be matched at the boundaries unless the original chain already has only next-neighbor transitions at 0 and N.

Despite this drawback this approximation should yield good results for "slowly moving" processes, i.e. processes where large jumps are rare. The

heuristics reason for this is that the first two moments of the approximating process are the same as those in the original chain at each interior state. If large jumps are rare, then the higher order moments on the two chains will also be close.

By far the biggest advantage of the birth-death approximation is the ease with which the approximate stationary distribution can be calculated. For a b - d chain of birth parameters λ_i and death parameters μ_i, the stationary distribution $\underset{\sim}{\pi}$ is given by

$$\pi_i = \frac{p_i}{\Sigma p_i}$$

where

$$p_0 \overset{\Delta}{=} 1 \quad \text{and} \quad p_i \; \frac{\lambda_0\lambda_1\cdots\lambda_{i-1}}{\mu_1\mu_2\cdots\mu_i} \; .$$

Note that the simple form of $\underset{\sim}{\pi}$ also allows rapid calculation of the approximate expected queue length $\Sigma i\pi_i$ and the approximate expected length of a busy cycle $\frac{1}{\pi_0}$. This type of approximation also appears more promising in the area of error-bounds than a diffusion approximation.

APPLICATION TO SLOTTED-ALOHA

As was mentioned earlier, the birth-death approximation works well for "slowly moving" processes. The slotted-ALOHA is such a process because most of the probability in the transition matrix is concentrated on the meain-, super- and sub-diagonals, i.e. is almost a birth-death process.

It should be noted that the slotted-ALOHA allows transitions from state 0 to all states except state 1, which is a rather unfortunate

situation since the approximation allows transitions only to state 1. Although this does not pose a major problem (because P_{00}'s large), it does cause the approximate stationary probability of state 1 to be too large.

It is not too difficult to calculate the stationary distribution of an "altered" approximation which allows the transition probabilities from one state, in this case state 0, to be equal to the actual transition probabilities, while approximating the rest of the chain with a b - d process. This, however, makes the form of the stationary distribution much more complicated and thus destroys the simplicity of the approximation.

Tables 1, 2, and 3 present examples of computations, where

- APPX1 - simple birth-death approximation
- APPX2 - altered birth-death approximation
- EXACT - exact stationary distribution (calculated via P^n $n \to \infty$)
- SHORT - exact stationary distribution (calculated via $\pi R P^{P^n}$ $n \to \infty$ see below)

As these computations show, the approximation is good although the relative error is fairly large for those states whose exact stationary distribution is less than 10^{-5}. The absolute errors are small. It is important to note that the approximate distributions have their relative maxima and minima at the same states.

SOME SIDE NOTES IN THE COMPUTATIONS

It is, of course, possible to calculate the exact stationary distribution of a Markov chain by repeatedly squaring the transition matrix. This is tantamount to grinding out P^n $n \to \infty$. This type of calculational bullying has

several drawbacks. First of all, it doesn't lend any insight into the possible changes in performance which would result from small changes in the parameters. It also is subject to roundoff errors since the number of necessary computations is large, as is the range of values. Lastly, the "brute-force" method is wasteful of computer time when applied to large chains. The computations of the exact disk given here required 27 CPU seconds, whereas the approximation required less than three.

As is noted, another use of the b-d approximation is in calculating the exact distribution. This can be done by calculating successive values of πP^n. Although this is also subject to roundoff error, it requires far less time thatn the brute-force method - the calculations here (SHORT) required 20 CPU sec.

CONCLUSION

Taking Nelson's lead, it appears that approximations of Markov chains are possible and useful. It also appers that a birth-death approximation though less elegant, is easier to use and just as accurate as a diffusion approximation. The direction of the research at this time is two fold, namely error bounds for the birth-death approximation are being computed with the end of perturbation theory, and the shape preserving properties of the approximation are being investigated.

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TABLE 1

CALC'F0=.003 F1=.06'

STATE	AFFX1	AFFX2	LOG AFFX1	LOG AFFX2
0	0.000029	0.000055	¯4.543581	¯4.259922
1	0.000039	0.000023	¯4.403736	¯4.640807
2	0.000041	0.00004	¯4.384059	¯4.399743
3	0.000037	0.000037	¯4.435848	¯4.437015
4	0.000029	0.000029	¯4.534418	¯4.5345
5	0.000022	0.000022	¯4.665206	¯4.665214
6	0.000015	0.000015	¯4.818784	¯4.818788
7	0.00001	0.00001	¯4.988653	¯4.988657
8	0.000007	0.000007	¯5.170116	¯5.17012
9	0.000004	0.000004	¯5.359656	¯5.35966
10	0.000003	0.000003	¯5.554563	¯5.554566
11	0.000002	0.000002	¯5.752698	¯5.752701
12	0.000001	0.000001	¯5.952341	¯5.952344
13	0.000001	0.000001	¯6.152083	¯6.152087
14	0.	0.	¯6.350758	¯6.350761
15	0.	0.	¯6.547382	¯6.547386
16	0.	0.	¯6.741122	¯6.741125
17	0.	0.	¯6.931261	¯6.931265
18	0.	0.	¯7.117181	¯7.117184
19	0.	0.	¯7.298341	¯7.298345
20	0.	0.	¯7.474269	¯7.474273
21	0.	0.	¯7.644547	¯7.64455
22	0.	0.	¯7.808803	¯7.808806
23	0.	0.	¯7.966707	¯7.966711
24	0.	0.	¯8.117964	¯8.117968
25	0.	0.	¯8.262309	¯8.262313
26	0.	0.	¯8.399504	¯8.399508
27	0.	0.	¯8.529333	¯8.529336
28	0.	0.	¯8.651602	¯8.651606
29	0.	0.	¯8.766136	¯8.76614
30	0.	0.	¯8.872776	¯8.87278
31	0.	0.	¯8.97138	¯8.971383
32	0.	0.	¯9.061817	¯9.06182
33	0.	0.	¯9.143971	¯9.143975
34	0.	0.	¯9.217739	¯9.217742
35	0.	0.	¯9.283026	¯9.283029
36	0.	0.	¯9.339749	¯9.339753
37	0.	0.	¯9.387836	¯9.38784
38	0.	0.	¯9.427223	¯9.427226
39	0.	0.	¯9.457854	¯9.457857
40	0.	0.	¯9.479682	¯9.479686
41	0.	0.	¯9.492671	¯9.492675
42	0.	0.	¯9.496789	¯9.496793
43	0.	0.	¯9.492014	¯9.492018
44	0.	0.	¯9.478332	¯9.478336
45	0.	0.	¯9.455735	¯9.455739
46	0.	0.	¯9.424225	¯9.424229

TABLE 1. (con't)

47	0.	0.	¯9.383809	¯9.383813
48	0.	0.	¯9.334503	¯9.334507
49	0.	0.	¯9.276331	¯9.276335
50	0.	0.	¯9.209324	¯9.209328
51	0.	0.	¯9.133522	¯9.133526
52	0.	0.	¯9.048972	¯9.048976
53	0.	0.	¯8.955731	¯8.955734
54	0.	0.	¯8.853863	¯8.853866
55	0.	0.	¯8.743442	¯8.743445
56	0.	0.	¯8.624552	¯8.624555
57	0.	0.	¯8.497286	¯8.49729
58	0.	0.	¯8.361749	¯8.361752
59	0.	0.	¯8.218055	¯8.218058
60	0.	0.	¯8.066331	¯8.066335
61	0.	0.	¯7.906717	¯7.906721
62	0.	0.	¯7.739365	¯7.739369
63	0.	0.	¯7.564443	¯7.564447
64	0.	0.	¯7.382133	¯7.382137
65	0.	0.	¯7.192635	¯7.192639
66	0.	0.	¯6.996165	¯6.996169
67	0.	0.	¯6.792961	¯6.792964
68	0.	0.	¯6.58328	¯6.583284
69	0.	0.	¯6.367405	¯6.367408
70	0.000001	0.000001	¯6.145642	¯6.145645
71	0.000001	0.000001	¯5.918327	¯5.91833
72	0.000002	0.000002	¯5.685826	¯5.68583
73	0.000004	0.000004	¯5.448541	¯5.448544
74	0.000006	0.000006	¯5.20691	¯5.206914
75	0.000011	0.000011	¯4.961417	¯4.961421
76	0.000019	0.000019	¯4.712592	¯4.712595
77	0.000035	0.000035	¯4.461018	¯4.461021
78	0.000062	0.000062	¯4.207342	¯4.207345
79	0.000112	0.000112	¯3.952278	¯3.952282
80	0.000201	0.000201	¯3.696622	¯3.696626
81	0.000362	0.000362	¯3.441258	¯3.441262
82	0.00065	0.00065	¯3.187177	¯3.187181
83	0.00116	0.00116	¯2.935493	¯2.935496
84	0.002054	0.002054	¯2.68746	¯2.687464
85	0.003593	0.003593	¯2.444506	¯2.444509
86	0.006191	0.006191	¯2.208256	¯2.20826
87	0.010457	0.010457	¯1.980583	¯1.980587
88	0.017232	0.017232	¯1.763654	¯1.763658
89	0.027542	0.027542	¯1.560003	¯1.560007
90	0.042401	0.0424	¯1.372629	¯1.372632
91	0.062356	0.062356	¯1.205119	¯1.205123
92	0.086728	0.086727	¯1.061841	¯1.061845
93	0.112665	0.112664	¯0.948211	¯0.948215
94	0.134552	0.134551	¯0.87111	¯0.871114
95	0.144691	0.14469	¯0.839559	¯0.839563
96	0.136181	0.13618	¯0.865884	¯0.865888
97	0.107662	0.107661	¯0.967936	¯0.96794
98	0.067004	0.067004	¯1.173899	¯1.173902
99	0.029167	0.029166	¯1.535114	¯1.535118
100	0.006657	0.006656	¯2.176752	¯2.176756

TABLE 2.

USING THE APPROXIMATION TO START.......
AFTER LOOPING 290 TIMES.......

STATE	SHORT	LOG SHORT
0	0.000044	¯4.360016
1	0.000036	¯4.444273
2	0.00004	¯4.396149
3	0.000033	¯4.484199
4	0.000026	¯4.582656
5	0.000019	¯4.712498
6	0.000014	¯4.857873
7	0.00001	¯5.016323
8	0.000007	¯5.183849
9	0.000004	¯5.357937
10	0.000003	¯5.536503
11	0.000002	¯5.717892
12	0.000001	¯5.90074
13	0.000001	¯6.083908
14	0.000001	¯6.266431
15	0.	¯6.447484
16	0.	¯6.626352
17	0.	¯6.802414
18	0.	¯6.975129
19	0.	¯7.144026
20	0.	¯7.308692
21	0.	¯7.468773
22	0.	¯7.623961
23	0.	¯7.773998
24	0.	¯7.918664
25	0.	¯8.05778
26	0.	¯8.191196
27	0.	¯8.318791
28	0.	¯8.440462
29	0.	¯8.556116
30	0.	¯8.665664
31	0.	¯8.769004
32	0.	¯8.866019
33	0.	¯8.956558
34	0.	¯9.040433
35	0.	¯9.117409
36	0.	¯9.187199
37	0.	¯9.249468
38	0.	¯9.303837
39	0.	¯9.349894
40	0.	¯9.387217
41	0.	¯9.415392
42	0.	¯9.434043
43	0.	¯9.442855
44	0.	¯9.441594
45	0.	¯9.430119
46	0.	¯9.408383

TABLE 2 (con't)

47	0.	¯9.376424
48	0.	¯9.33435
49	0.	¯9.282317
50	0.	¯9.22051
51	0.	¯9.149122
52	0.	¯9.068343
53	0.	¯8.97835
54	0.	¯8.879304
55	0.	¯8.771348
56	0.	¯8.654615
57	0.	¯8.529226
58	0.	¯8.3953
59	0.	¯8.252957
60	0.	¯8.102321
61	0.	¯7.943527
62	0.	¯7.776723
63	0.	¯7.602072
64	0.	¯7.419753
65	0.	¯7.229969
66	0.	¯7.032939
67	0.	¯6.82891
68	0.	¯6.61815
69	0.	¯6.400957
70	0.000001	¯6.177656
71	0.000001	¯5.948604
72	0.000002	¯5.714193
73	0.000003	¯5.474849
74	0.000006	¯5.231043
75	0.00001	¯4.983287
76	0.000019	¯4.732144
77	0.000033	¯4.478234
78	0.00006	¯4.222236
79	0.000108	¯3.9649
80	0.000196	¯3.707055
81	0.000355	¯3.449621
82	0.00064	¯3.19362
83	0.001148	¯2.940194
84	0.002039	¯2.690626
85	0.003578	¯2.446364
86	0.006179	¯2.209051
87	0.010458	¯1.98057
88	0.017255	¯1.763089
89	0.027597	¯1.559138
90	0.042492	¯1.371695
91	0.062471	¯1.20432
92	0.086829	¯1.061334
93	0.112696	¯0.948091
94	0.134465	¯0.871391
95	0.144493	¯0.840153
96	0.135964	¯0.866575
97	0.107559	¯0.968352
98	0.067068	¯1.173484
99	0.029303	¯1.533092
100	0.006728	¯2.172089

TABLE 3

AFTER STRAIGHTFORWARD BUT TEDIOUS CALCULATIONS,
(RAISING P TO THE 1.0737E9TH POWER).....

STATE	EXACT	LOG EXACT
0	0.000018	¯4.746165
1	0.000015	¯4.830447
2	0.000017	¯4.782327
3	0.000013	¯4.870391
4	0.000011	¯4.968858
5	0.000008	¯5.09871
6	0.000006	¯5.244095
7	0.000004	¯5.402554
8	0.000003	¯5.570088
9	0.000002	¯5.74418
10	0.000001	¯5.922746
11	0.000001	¯6.104126
12	0.000001	¯6.286952
13	0.	¯6.47008
14	0.	¯6.652537
15	0.	¯6.833483
16	0.	¯7.012188
17	0.	¯7.18801
18	0.	¯7.360379
19	0.	¯7.528786
20	0.	¯7.692772
21	0.	¯7.851922
22	0.	¯8.00586
23	0.	¯8.154241
24	0.	¯8.29675
25	0.	¯8.433095
26	0.	¯8.563009
27	0.	¯8.686246
28	0.	¯8.802574
29	0.	¯8.911784
30	0.	¯9.013677
31	0.	¯9.108071
32	0.	¯9.194797
33	0.	¯9.273698
34	0.	¯9.34463
35	0.	¯9.407459
36	0.	¯9.462062
37	0.	¯9.508326
38	0.	¯9.54615
39	0.	¯9.57544
40	0.	¯9.596114
41	0.	¯9.608097
42	0.	¯9.611324
43	0.	¯9.605742
44	0.	¯9.591302
45	0.	¯9.567969
46	0.	¯9.535715

TABLE 3 (con't)

47	0.	¯9.49452
48	0.	¯9.444377
49	0.	¯9.385285
50	0.	¯9.317255
51	0.	¯9.240307
52	0.	¯9.154472
53	0.	¯9.059791
54	0.	¯8.956317
55	0.	¯8.844112
56	0.	¯8.723252
57	0.	¯8.593824
58	0.	¯8.455929
59	0.	¯8.30968
60	0.	¯8.155206
61	0.	¯7.992648
62	0.	¯7.822167
63	0.	¯7.643936
64	0.	¯7.458148
65	0.	¯7.265017
66	0.	¯7.064773
67	0.	¯6.857671
68	0.	¯6.643989
69	0.	¯6.424029
70	0.000001	¯6.198121
71	0.000001	¯5.966627
72	0.000002	¯5.729939
73	0.000003	¯5.488486
74	0.000006	¯5.242736
75	0.00001	¯4.993203
76	0.000018	¯4.740447
77	0.000033	¯4.485083
78	0.000059	¯4.227789
79	0.000107	¯3.969309
80	0.000195	¯3.710466
81	0.000353	¯3.452174
82	0.000638	¯3.195448
83	0.001144	¯2.941423
84	0.002035	¯2.691373
85	0.003575	¯2.446737
86	0.006178	¯2.209148
87	0.01046	¯1.980478
88	0.017263	¯1.762886
89	0.027613	¯1.558887
90	0.042516	¯1.371449
91	0.062501	¯1.204116
92	0.086857	¯1.061197
93	0.112712	¯0.94803
94	0.134462	¯0.8714
95	0.144475	¯0.840208
96	0.135946	¯0.866635
97	0.107557	¯0.968359
98	0.067087	¯1.173359
99	0.029327	¯1.532737
100	0.006739	¯2.171384

END

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